IMAGE COMPLEXITY AND SPATIAL INFORMATION

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ABSTRACT

The complexity of an image tells many aspects of the image content and is an important factor in the selection of source material for testing various image processing methods. We explore objective measures of complexity that are based on compression. We show that spatial information (SI) measures strongly correlate with compression-based complexity measures. Among the commonly used SI measures, the mean of the edge magnitude is shown to be the best predictor. Moreover, we find that compression-based complexity of an image normally increases with decreasing resolution.

***Index Terms*—** Image quality, image compression, Kolmogorov complexity, SI, resolution

1. INTRODUCTION

The knowledge of image complexity is useful in many applications. It can be used to determine the compression level and bandwidth allocation, as an image with low complexity can be compressed more easily and requires less bandwidth than an image with high complexity [1]. Moreover, complexity-based similarity measures are used in many high-level image understanding and recognition problems, such as content-based image retrieval (CBIR) [2], image clustering and classification [3], as well as aesthetic classification [4, 5]. Last but not least, image complexity is an important factor in the design of image and video quality databases [6].

Yet the definition of the complexity of an image is not as straightforward as it seems. Researchers from various fields have proposed different measures to estimate image complexity. In [1, 7, 8], observers were asked to rate the perceived complexity of images. Despite a high correlation with human perception [9], such subjective rating scores are costly to obtain, less consistent, and not necessarily relevant, as subjective image complexity may not be the same as objective complexity. Thus, objective measures of image complexity are much needed. In the literature, fuzzy approaches [10] and independent component analysis (ICA) [2] have been proposed to determine the complexity of an image. Compression-based image complexity, which originates from the notion of Kolmogorov complexity [2, 3, 11], has attracted increasing attention due to its strong information theoretic justification. In this paper, we review existing and propose new compression-based measures of image complexity.

Oftentimes, engineers would like to know the complexity of an image before compressing it so as to determine the optimal tradeoff between image compression and image quality. One way to get such information, which has to be extremely fast to compute, is to measure the spatial information (SI) contained in the image. In this paper, we examine the relationship between common SI measures and compression-based image complexity measures, which to our knowledge has not been done before, thus enabling researchers to make an informed decision on which SI measures to use. We consider only grayscale images in order to eliminate influences from color. Finally, we examine the effect of resolution change on image complexity and SI, showing that both normally increase as resolution decreases. We also offer spatial-frequency domain explanations for this behavior.

The rest of the paper is organized as follows. Section 2 explains the concept of Kolmogorov complexity, introduces existing compression-based measures of image complexity, proposes new measures, and studies their correlations. Section 3 defines the commonly used SI measures and investigates their correlation with image complexity. Section 4 examines the resolution dependence of image complexity using spatial-frequency analysis. Section 5 concludes the paper with possible directions to future work.

2. COMPRESSION-BASED IMAGE COMPLEXITY

In Shannon's information theory, entropy is used to measure the amount of information in a set of symbols such as an image [12]. However, it is not a good measure for image complexity, because entropy is calculated without considering spatial structures. For instance, the two binary images in Fig. 1 both have an entropy of 1, but image 1(b) is clearly much more complex than 1(a).

Hence, people turn to algorithmic information theory for a suitable complexity measure [11, 13]. Kolmogorov defines the complexity of an object to be the length of the shortest

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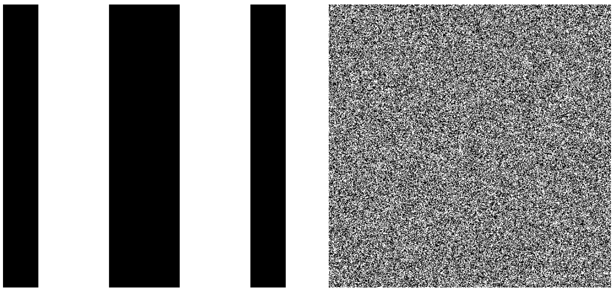


(a) (b)

Fig. 1: Two images with the same entropy of 1.

binary computer program that describes it [12]. However, the Kolmogorov complexity is not computable. Thus, we have to approximate Kolmogorov complexity with a standard real-world compressor [11].

We first define the compression ratio as follows:

$$CR = \frac{s(I)}{s(C(I))}, \tag{1}$$

where $s(I)$ is the file size of the uncompressed (grayscale) image I, and $s(C(I))$ is the file size of the output of compressor C.

In the development of complexity-based similarity metrics [2, 3, 11], lossless compression is used as a complexity-based feature. Here, we define the first image complexity (IC) measure as the inverse of the lossless compression ratio of the image:

$$IC_{LS} = \frac{1}{CR}, \tag{2}$$

where 'LS' stands for lossless.

In computing aesthetics [4, 5], lossy compression and distortion are used to define image complexity:

$$IC_{RMSE}(q) = \frac{RMSE(q)}{CR(q)}, \tag{3}$$

where $RMSE$ is the root-mean-square error between the original image and the lossy compressed image, and q is a parameter that controls the amount of quantization in lossy compression; for example, $q \in \{1, 2, \ldots, 100\}$ in JPEG and JPEG2000 compression, where higher q values correspond to lighter compression and better image quality.

We propose a third compression-based definition, which is also based on lossy compression, but without the error term, because the compression ratio by itself indicates how difficult it is to compress an image:

$$IC_{LY}(q) = \frac{1}{CR(q)}, \tag{4}$$

where 'LY' stands for lossy compression.

Both complexity measures IC_{RMSE} and IC_{LY} are functions of the compression quality factor q. As shown in Fig. 2, when applied to an image that is compressed using JPEG or JPEG2000, IC_{LY} is a monotonically increasing function of q, while IC_{RMSE} is not a monotonic function, because $RMSE$ and CR increase differently as q decreases.

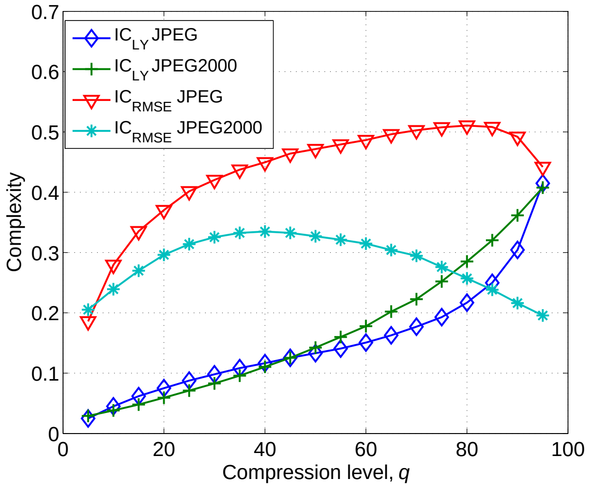


Fig. 2: Complexity measures as a function of compression level q for JPEG and JPEG2000 compression.

We would also like to know the correlation among different complexity measures at different quality factors q. To test this, we use the reference images of the CSIQ database [14], comprising 30 uncompressed images from five categories, namely Animals, Landscape, People, Plants, and Urban. The images are of size 512×512 and were converted to grayscale to eliminate the effect of color on compression. We test $q = 25$ and $q = 75$ to cover different compression levels. The result for JPEG-based complexity measures is shown in Table 1. The correlation between any pair of complexity measures is above 0.91. Moreover, the correlations between pairs $(IC_{RMSE}(25), IC_{RMSE}(75))$ and $(IC_{LY}(25), IC_{LY}(75))$ are both above 0.98. We observe the same high correlations between other quality settings as well.

Table 1: Correlation among different complexity measures.

	IC_{LS}	$IC_{RMSE}(25)$	$IC_{RMSE}(75)$	$IC_{LY}(25)$	$IC_{LY}(75)$
IC_{LS}	1				
$IC_{RMSE}(25)$	0.9213	1			
$IC_{RMSE}(75)$	0.9167	0.9817	1		
$IC_{LY}(25)$	0.9176	0.9795	0.9396	1	
$IC_{LY}(75)$	0.9501	0.9827	0.9685	0.9885	1

As mentioned earlier, Kolmogorov complexity is not computable, which makes complexity measures compressor-dependent. Nevertheless, we expect to see a high correlation between complexity measures based on different compression methods. We test this hypothesis using both JPEG and JPEG2000 compression for all three complexity measures (JPEG-LS is used for JPEG lossless compression). The results in Fig. 3 show a near-perfect match between them. The corresponding correlation coefficients are 0.9912, 0.9877, and 0.9509 for IC_{LS}, IC_{LY}, and IC_{RMSE} respectively (the last being somewhat lower mainly due to the slightly non-linear relationship).

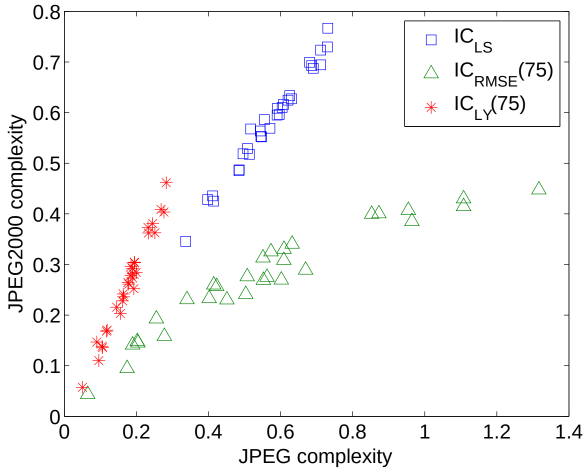


Fig. 3: Correlation between JPEG and JPEG2000 complexity measures.

3. SPATIAL INFORMATION

Spatial information (SI) is an indicator of edge energy [15] and has been commonly used as the basis for estimating image complexity. Let s_h and s_v denote gray-scale images filtered with horizontal and vertical Sobel kernels, respectively.

$$SI_r = \sqrt{s_h^2 + s_v^2} \tag{5}$$

represents the magnitude of spatial information at every pixel. The SI measures commonly used to characterize image complexity are mean, root-mean-square, and standard deviation of the SI_r values across all the pixels in the image [15]. These are mathematically expressed as:

$$SI_{\text{mean}} = \frac{1}{P}\sum SI_r, \tag{6}$$

$$SI_{\text{rms}} = \sqrt{\frac{1}{P}\sum SI_r^2}, \text{ and} \tag{7}$$

$$SI_{\text{stdev}} = \sqrt{\frac{1}{P}\sum SI_r^2 - SI_{\text{mean}}^2}, \tag{8}$$

where P is the number of pixels in the image. These SI measures are fast to compute and used to predict the complexity of images.

We examine the correlations between each SI measure and JPEG-based image complexity measures, using again the 30 reference images from the CSIQ image database. Fig. 4 illustrates that SI_{mean} is a significantly better predictor than the other two SI measures, regardless of the complexity measures considered. This is further quantified with the correlation coefficients shown in Table 2, which shows SI_{mean} to be better than SI_{rms}, and SI_{stdev} being the worst by far in predicting image complexity. The same trends are observed for JPEG2000-based complexity measures as well as other compression levels.

Table 2: Correlation coefficients between SI measures and JPEG-based complexity measures.

	IC_{LS}	$IC_{RMSE}(75)$	$IC_{LY}(75)$
SI_{mean}	0.9104	0.9352	0.9720
SI_{rms}	0.8454	0.9183	0.9368
SI_{stdev}	0.7197	0.8341	0.8350

SI measures are also relatively robust to compression. Fig. 5 shows SI as a function of q for a typical image from the CSIQ database under JPEG compression. All three SI measures exhibit very little variation across a wide range of compression levels. In particular, SI_{mean} is nearly constant except for unusually heavy compression ($q < 20$). This behavior makes SI well suited for image activity characterization and content classification in image and video quality assessment applications, and a number of video quality metrics use SI or closely related measures for this purpose [16–18].

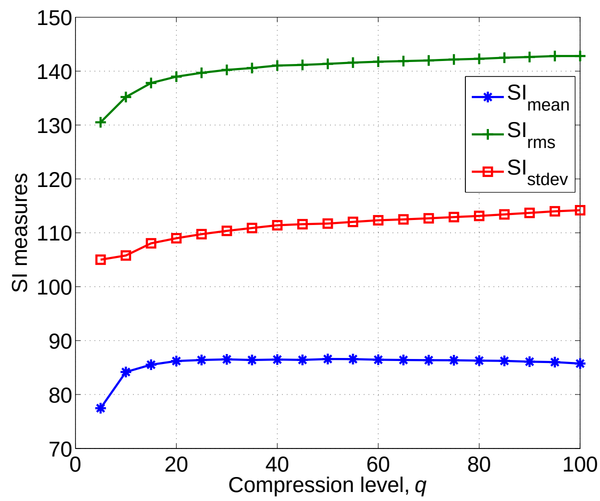
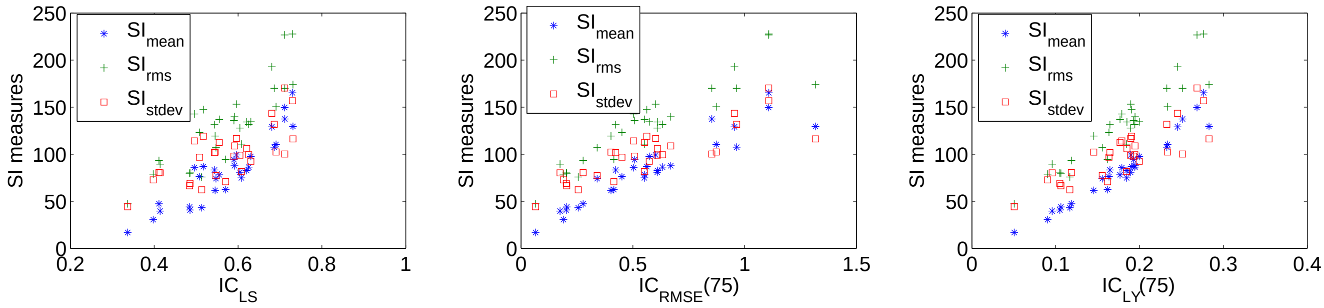


Fig. 5: SI measures as a function of compression level q for JPEG compression.



(a) (b) (c)

Fig. 4: SI measures versus various JPEG-based complexity measures.

4. EFFECTS OF RESOLUTION CHANGES

The majority of the images in the CSIQ database exhibit higher complexity as the image resolution decreases. To further confirm this trend for heavy reductions, we also test on images obtained from the Digital Photography Review web site (http://www.dpreview.com/). These images were taken with Nikon D600/D800 and Canon EOS 5D cameras and are of very high quality and resolution (21-36 megapixels). We reduce the resolution of the images by different integer factors (a low pass filter is applied prior to subsampling to prevent aliasing). The result for one high-resolution image, 7(a), is shown in Fig. 6. This negative correlation of complexity measures and spatial information with image resolution can be observed for the majority of our test images.

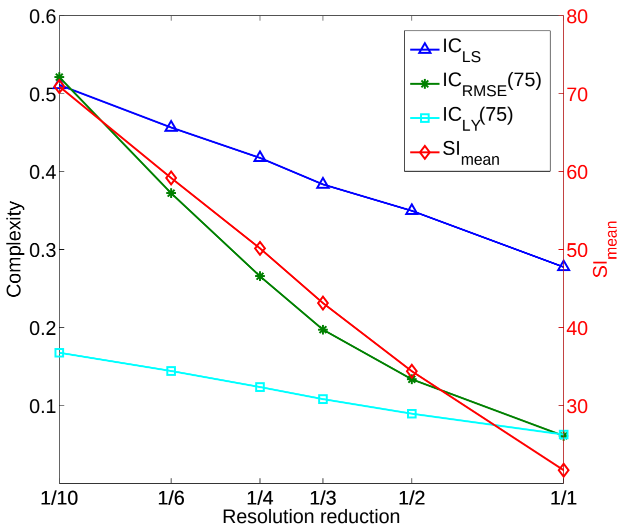


Fig. 6: Complexity measures at different image resolutions.

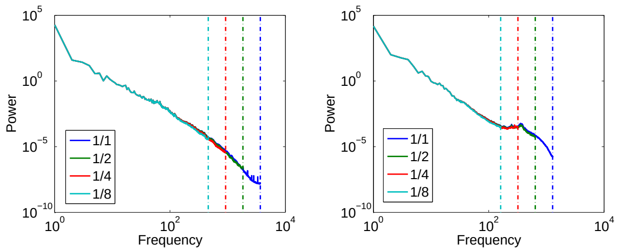
For most natural images, energy is concentrated in the low-frequency components comprised of homogeneous image patches. Intuitively, as image resolution decreases, large patches become smaller and small patches become localized features, such as lines, edges or corners, and so this creation of new localized features surpasses the high frequency loss from reducing resolution of existing localized features.

However, we also observed a few images with the opposite behavior, i.e. lower complexity at reduced resolution, for instance image 7(b). These images contain a relatively large portion of fine-scaled localized features (fabric textures in all our examples), which are lost when the image is subsampled, and the creation of new localized features is unable to make up for that.

(a) (b)

Fig. 7: High resolution images: (a) negative correlation between complexity and resolution, (b) positive correlation between complexity and resolution.

To support the aforementioned argument, we plot the radially averaged power spectrum (RAPS) for images 7(a) and 7(b). We also observe similar patterns in RAPS plots for other test images. RAPS is a convenient way to visualize direction-independent frequency energy in a 1-D plot and has been used in rotation and scale invariant texture analysis [19]. In Fig. 8a, the RAPS for images with a negative correlation between complexity and resolution has a nearly log linear decaying high frequency energy, which is indeed characteristic of most natural images [20], whereas in Fig. 8b, there is a sudden surge in high frequency energy, which would be lost with resolution reduction and thus cause the complexity to decrease.



(a) (b)

Fig. 8: Radially averaged power spectrum for two types of images: (a) negative and (b) positive correlation between complexity and resolution. The dash-dotted lines indicate the highest frequency at the respective resolutions.

5. CONCLUSION

This paper makes three main contributions.

1. We have studied three compression-based image complexity measures, namely IC_{LS}, $IC_{RMSE}(q)$, and $IC_{LY}(q)$. Although these measures depend on the specific image compressor used, we have demonstrated nearly perfect correlations between JPEG and JPEG2000-based complexity measures. We have also shown that complexity measures generated by different compression quality factors q are highly correlated. Moreover, there exists a strong correlation among IC_{LS}, $IC_{RMSE}(q)$, and $IC_{LY}(q)$ for a given image compressor and compression level.

2. We have evaluated the correlation between different SI measures and complexity measures, showing that SI_{mean} is the best predictor of image complexity among the three SI measures considered. It is also robust to image compression.

3. We have found a general trend between image complexity and image resolution, i.e. image complexity generally increases as image resolution decreases, except for images with a large portion of fine-grained textures.

We plan to extend this work to color images as well as video, for which there is less agreement as to how complexity can be estimated. The question is also how the contributions of spatial, temporal, and color activity can be best be measured individually and combined.

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